Guess Paper – 2010
Class – XII
Subject - MATHEMATICS

Time Allowed : 3 hours
Maximum Marks : 100

<u>General Instructions:</u>

The question paper consists of 29 questions divided into three sections A, B & C. Section A comprises of 10 questions of 1 mark each, section B comprises of 12 questions of 4 marks each & section C comprises of 7 questions of 6 marks each.

<u>SECTION – A</u>

1. Write the identity element for the binary operation * defined on the set R of real numbers by the rule a * b = $\frac{3ab}{8}$, for all a, b $\in$ R.

2. If $A = \begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$. Write the order of AB and BA.

3. What is the principal value of $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) + \cos^{-1}\left(\cos\frac{\pi}{6}\right)$?

4. Evaluate : $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$

5. If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, evaluate $F(0)$.

6. Write the value of $\int_0^{\pi/2} \log\left(\frac{3+5\cos x}{3+5\sin x}\right) dx$.

7. Find the angle made by the vectors $\hat{i} - \hat{j}$ with the y-axis.

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8. Find the value of k for which the lines $\frac{x-1}{3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{6-z}{5}$ are perpendicular to each other.

9. Write the value of $\int e^{3\log x}.(x^4)\,dx$.

10. Find $\left|\vec{a}\times\vec{b}\right|$, if $\vec{a}=\hat{i}-7\hat{j}+7\hat{k}$ and $\vec{b}=3\hat{i}-2\hat{j}+2\hat{k}$

SECTION – B

11. Find the equation of the normals to the curve $y=x^3+2x+6$, which are parallel to the line $x+14y+4=0$.

OR

Find the intervals in which the function f given by $f(x)=\sin x+\cos x,\ 0\le x\le 2\pi$ is strictly increasing or strictly decreasing.

12. If $y=\left(\tan^{-1}x\right)^2$, show that $\left(x^2+1\right)^2 y_2+2x\left(x^2+1\right)y_1=2$.

13. Using properties of determinants, prove that :

$$\begin{vmatrix} x & x^2 & 1+px^3 \\ y & y^2 & 1+py^3 \\ z & z^2 & 1+pz^3 \end{vmatrix}=(1+pxyz)(x-y)(y-z)(z-x)$$

OR

Using elementary transformations, find the inverse of the matrix $A=\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

14. Let A and B be sets. Show that $f : A\times B\to B\times A$ such that $f(a,b)=(b,a)$ is bijective.

15. Two cards are drawn simultaneously (or successively without replacement) from a well shuffled pack of 52 cards. Find the mean, variance and standard deviation of the number of kings.

16. Evaluate : $\int_0^{\pi/4}\frac{\sin x+\cos x}{9+16\sin 2x}\,dx$.

OR

Prove that : $\int_0^{\pi} \frac{x}{1+\sin x}\, dx = \pi$

17. If $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$, then find the value of x.

18. If $x = a\left[\cos t + \log\left|\tan\frac{t}{2}\right|\right]$ and then $y = a\sin t$ find $\frac{dy}{dx}$ at $t = \frac{\pi}{4}$.

19. Find the distance of the point $(-1,-5,-10)$ from the point of intersecting of the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$.

20. Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k}$, $\vec{b} = \hat{i} - 4\hat{j} + 5\hat{k}$, $\vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and is such that $\vec{d}\cdot\vec{c} = 21$.

OR

If with reference to the right handed system of mutually perpendicular unit vectors $\hat{i}$, $\hat{j}$ and $\hat{k}$, $\vec{\alpha} = 3\hat{i} - \hat{j}$, $\vec{\beta} = 2\hat{i} + \hat{j} - 3\hat{k}$ then express $\vec{\beta}$ in the form of $\vec{\beta} = \vec{\beta}_1 + \vec{\beta}_2$, where $\vec{\beta}_1$ is parallel to $\vec{\alpha}$ and $\vec{\beta}_2$ is perpendicular to $\vec{\alpha}$.

21. Solve : $xdy - ydx = \sqrt{x^2 + y^2}\,dx$

22. Solve the differential equation: $\left(1 + x^2\right)\frac{dy}{dx} + y = \tan^{-1} x$

<u>SECTION – C</u>

23. If $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find $(AB)^{-1}$.

24. Find the image of the point (1, 2, 3) in the plane $x + 2y + 4z = 38$.

25. Find the area between the curves $y = x^2$ and $y = x$.

26. An aeroplane can carry a maximum of 200 passengers. A profit of Rs 1000 is made on each executive class ticket and a profit of Rs 600 is made on each economy class ticket. The airline reserves at least 20 seats for executive class. However, at least 4 times as many passengers prefer to travel by economy class than by the executive class. Determine how many tickets of each type must be sold in order to maximise the profit for the airline. What is the maximum profit?

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27. Prove that the volume of the largest cone that can be inscribed in a sphere of radius R is $\frac{8}{27}$ of the volume of the sphere.

OR

A window is in the form of a rectangle surmounted by a semicircular opening. The total perimeter of the window is 10 m. Find the dimensions of the window to admit maximum light through the whole opening.

28. A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}, \frac{1}{5}, \frac{1}{10}$ and $\frac{2}{5}$. The probabilities that he will be late are $\frac{1}{4}, \frac{1}{3}$ and $\frac{1}{12}$, if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. When he arrives, he is late. What is the probability that he comes by train?

29. Evaluate : $\int_0^{\pi/2} \frac{x}{\sin x + \cos x}\, dx.$

OR

Evaluate : $\int_0^1 \sin^{-1} \sqrt{\frac{x}{a+x}}\, dx.$