Guess Paper – 2010
Class – XII
Subject – Maths

Time Allowed : 3 hours
Maximum Marks : 100

General Instructions:

The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 7 questions of six marks each.

SECTION – A

1. Evaluate : $\tan\left(\frac{1}{2}\cos^{-1}\left(-\frac{\sqrt{5}}{3}\right)\right)$.

2. Evaluate : $\int \frac{dx}{x\cos^2(1+\log x)}$.

3. Write the value of $\int_0^{\pi/2} \log\left(\frac{3+5\cos x}{3+5\sin x}\right) dx$.

4. If $f(x)=\sqrt{x},\ (x>0)$ and $g(x)=x^2-1$, find *fog* and *gof*.

5. If $F(x)=\begin{bmatrix}\cos x & -\sin x & 0\\ \sin x & \cos x & 0\\ 0 & 0 & 1\end{bmatrix}$, evaluate $F(0)$.

6. If $\vec{a}=\hat{i}+\hat{j}+\hat{k}$; $\vec{b}=2\hat{i}-\hat{j}+3\hat{k}$ and $\vec{c}=\hat{i}-2\hat{j}+\hat{k}$, find a unit vector parallel to the vector $2\vec{a}-\vec{b}+3\vec{c}$.

7. Find the value of k for which the lines $\frac{x-1}{-3}=\frac{y-2}{2k}=\frac{z-3}{2}$ and $\frac{x-1}{3k}=\frac{y-1}{1}=\frac{6-z}{5}$ are perpendicular.

8. If $|\vec{a}|=2$; $|\vec{b}|=7$ and $\vec{a}\times\vec{b}=3\hat{i}+2\hat{j}+6\hat{k}$, find the angle between $\vec{a}$ and $\vec{b}$.

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9. Evaluate : $\begin{vmatrix} a & b & c \\ a+2x & b+2y & c+2z \\ x & y & z \end{vmatrix}$

10. Find the integral value(s) of x if $\begin{vmatrix} x^2 & x & 1 \\ 0 & 2 & 1 \\ 3 & 1 & 4 \end{vmatrix} = 28.$

SECTION – B

11. If $y = \left(\tan^{-1} x\right)^2$, show that $\left(x^2+1\right)^2 y_2 + 2x\left(x^2+1\right)y_1 = 2.$

12. Evaluate : $\displaystyle\int \frac{x+3}{\sqrt{5-4x-x^2}}\, dx$

OR

Evaluate : $\displaystyle\int \frac{\sin x + \cos x}{9 + 16\sin 2x} dx.$

13. Show that f : N→ N, given by $f(x) = \begin{cases} x+1, \text{ if } x \text{ is odd} \\ x-1, \text{ if } x \text{ is even} \end{cases}$

is both one-one and onto.

14. If $x^y + y^x + x^x = a^b$, find $\dfrac{dy}{dx}$.

OR

If $x = a\left[\cos t + \log\left|\tan\dfrac{t}{2}\right|\right]$ and then $y = a\sin t$ find $\dfrac{dy}{dx}$ at $t = \dfrac{\pi}{4}$.

15. Sand is pouring from a pipe at the rate of 12 cm^3/s. The falling sand forms a cone on the ground in such a way that the height of the cone is always one-sixth of the radius of the base. How fast is the height of the sand cone increasing when the height is 4 cm?

OR

The length x of a rectangle is decreasing at the rate of 3 cm/minute and the width y is increasing at the rate of 2cm/minute. When x =10cm and

y = 6cm, find the rates of change of (a) the perimeter and (b) the area of the rectangle.

16. Let $\vec{a} = 4\hat{i} + 5\hat{j} - \hat{k},\ \vec{b} = \hat{i} - 4\hat{j} + 5\hat{k},\ \vec{c} = 3\hat{i} + \hat{j} - \hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ and $\vec{b}$, and is such that $\vec{d} \cdot \vec{c} = 21$.

17. Prove that : $\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) = \frac{\pi}{2}$.

OR

Find the value of : $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$.

18. Find the distance of the point $(-1,-5,-10)$ from the point of intersecting of the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and the plane $\vec{r}.(\hat{i} - \hat{j} + \hat{k}) = 5$.

19. Form a differential equation representing the family of curves $y = e^x (a \cos x + b \sin x)$, by eliminating arbitrary constants a and b.

20. Find the particular solution of the differential equation :

$\frac{dy}{dx} + y \cot x = 2x + x^2 \cot x \ (x \neq 0)$, given that $y = 0$ when $x = \pi/2$.

21. Show that : $\begin{vmatrix} (b+c)^2 & ba & ca \\ ab & (c+a)^2 & cb \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$

22. Let X denote the number of hours you study on a Sunday. Also it is known that

$$P(X = x) = \begin{cases} 0.1, & \text{if } x = 0 \\ kx, & \text{if x = 1 or 2} \\ k(5-x), & \text{if x = 3 or 4} \\ 0, & \text{otherwise} \end{cases}$$

where k is a constant.

(a) Find the value of k.

(b) What is the probability that you study atleast two hours? Exacty tow hours? Atmost two hours?

SECTION – C

23. Using matrices solve the following system of equations:

$$\frac{2}{x}-\frac{3}{y}+\frac{3}{z}=10; \qquad \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=10 \quad \text{and} \quad \frac{3}{x}-\frac{1}{y}+\frac{2}{z}=13$$

OR

Obtain the inverse of the following matrix using elementary row operations:

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$$

24. Evaluate : $\int_0^{\pi} \log(1+\cos x)\, dx$

25. Using the method of integration, find the area of the region bounded by the lines 2x + y = 4, 3x – 2y = 6 and x – 3y + 5 = 0.

OR

Make a rough sketch of the region given below and find the area using the method of integration : $\{(x, y); 0 \le y \le x^2 + 3, 0 \le y \le 2x + 3, 0 \le x \le 3\}$.

26. Find the distance of the point A(–2, 3, –4) from the line $\frac{x+2}{3}=\frac{2y+3}{4}=\frac{3z+4}{5}$ measured parallel to the plane $4x+12y-3z+1=0$.

27. An open box with a square base is to be made out of a given quantity of sheet of area a^2. Show that the maximum capacity of the box is $\frac{a^3}{6\sqrt{3}}$.

OR

A water tank has the shape of an inverted right circular cone with its axis vertical and vertex lowermost. Its semi-vertical angle is $\tan^{-1}(0.5)$. Water is poured into it at a constant rate of 5 cubic metre per hour. Find the rate at which the level of the water is rising at the instant when the depth of water in the tank is 4 m.

28. A toy company manufactures two types of dolls, A and B. Market tests and available resources have indicated that the combined production level should not exceed 1200 dolls per week and the demand for dolls of type B is at most half of that for dolls of type A. Further, the production level of dolls of type A can exceed three times the production of dolls of other type by at most 600 units. If the company makes profit of Rs 12 and Rs 16 per doll respectively on dolls A and B, how many of each should be produced weekly in order to maximise the profit?

29. Assume that the chance of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga?
