Real numbers.

1) Find the HCF of the integers 455 and 42 using Euclid's division algorithm.
2) Prove that any positive odd integer is of the form $4p+1$ or $4p+3$, where p is an integer.
3) Use Euclid's division lemma to show that the square of any positive integer is either of the form $3p$ or $3p+1$, where p is an integer.
4) The numbers 1424, 2096, and 1844 when divided by a number N give the same remainder 10. Find the highest such number N.
5) Prove that for any prime number p, $\sqrt{p}$ is an irrational number.
6) Prove that there is no natural number n for which 4^n ends with the digit 0.
7) Find the highest number less than 1000 that is divisible by each of the numbers 2, 3, 4, 5, 6, and 7.
8) Prove that $\left[\sqrt{2}+\sqrt{7}\right]$ is irrational.

9) State without actual division, whether the decimal expansions of the following rational numbers are terminating or non-terminating repeating. Give reasons to justify your answer.

(a) $\frac{189}{160}$ (1 mark)

(b) $\frac{24}{2^3 \times 3^2 \times 5^2}$ (1 mark)

10) The decimal expansions of some real numbers are given below. For each number, state whether it is rational or irrational. If the number is rational, then assuming it to be of the form $\frac{p}{q}$, what can be said about the prime factors of its denominator? Give reasons to justify your answer.

(a) 34.98 (1 mark)

(b) 23.353355333555… (1 mark)

(c) $9.87\overline{34}$ (1 mark)